

Moments (Flat Planes And Non Diagonal Forces):

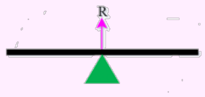
Types	Description

Definition: What Is A Moment?

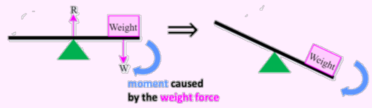
Let's first consider the following two examples to get some context.

Example 1: Seesaw

The whole plank is pivoting around the pivot.

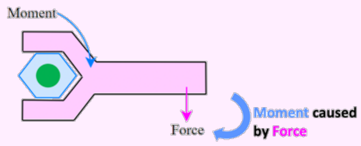


If we put a weight (some kind of force on a seesaw), this turns the seesaw i.e. the seesaw turns around the pivot. The turning force around the pivot point due to the force is called a moment. We would need another weight to keep the seesaw flat, meaning to keep it in equilibrium.



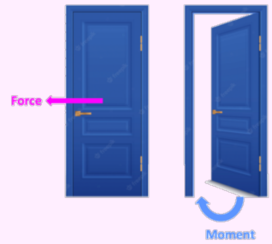
Example 2: Spanner Turning A Nut

If we apply a downwards force, the nut won't move downwards. It will instead turn clockwise. This turning effect is the moment.



Example 3: Hinge of a door

When we push open a door, we apply a force to the edge of the door furthest from its hinges. This effect has a turning effect on the door which is a moment which causes the door to rotate around its hinge



Now that you've seen three examples of a moment occurring, you should now be ready for the definition of a moment:

A force or system of forces may cause an object to turn. A moment is the turning effect of a force i.e. the moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis called a pivot. So, forces create moments that act around a pivot and the pivot is the point around which the object can rotate.

- On a seesaw, the pivot is the point in the middle
- On a spanner, the pivot in at the bolt
- On a door, the pivot is the hinge.

Key points to take away:

- A force applied to an object causes it to rotate and this rotational effect of a force is called a moment
- Moments can act about a point in a clockwise or anticlockwise direction and hence some moments are positive and some moments are negative
- Each force has a moment, so moments need to be calculated for every force. The above diagrams only have one force acting, hence only one moment
- The point chosen to take moments about (i.e. the pivot) could be any point on the body i.e. the middle, endpoints or wherever. In your course, you often choose the supports. How to know which point to take moments about will be explained in detail in this sheet in the step-by step-method section.

Assumptions

- The plank/rod/beam is **uniform** means that the weight acts in the centre
- The person/object is a **particle** means that the weight i.e. force acts where the person is standing/object is placed

Background Information

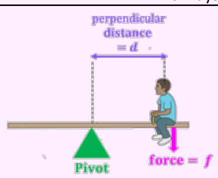
It is really important to realise that that there are 3 types of forces acting on a beam/rod:

- Forces in in x direction (parallel to the beam). These are the forces that we are used to
- Forces in in y direction (perpendicular to the beam). These are the forces that we are used to
- Moments.

This means that not only can we take **moments for the beam/rod**, but we can also **sum the forces** vertically and horizontally **for the whole system** as normal. So, there are 3 possible equations that one can form in order to find the unknowns asked for.

1) Consider the **whole system** and sum all forces **in the vertical direction and set equal to zero**. This is the method you're already familiar with when you set the sum of forces equal to ma, since $f = ma$ and here f is the sum of the forces in the vertical direction and $a = 0$
2) Consider the **whole system** and sum all forces **in the horizontal direction and set equal to zero**. This is the method you're already familiar with when you set the sum of forces equal to ma, since $f = ma$ and here f is the sum of the forces in the horizontal direction and $a = 0$
3) Consider the **beam/rod and take moments**: Sum of moments is always 0 (or the anti-clockwise moments equal the clockwise moments) if in equilibrium. For this course you will always be in equilibrium.

Formula And Detailed Method Of How To Calculate Moments

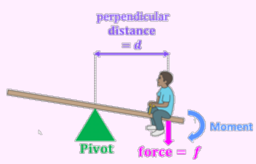


$$\text{Moment} = f\ d$$

f = force

d =perpendicular distance from the force to the pivot point

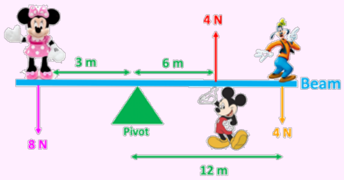
To calculate each moment, we just multiply each force by the perpendicular DISTANCE from the pivot point (the point we chose to take moments about) to the force



Moments are measured in newton-metres (Nm), forces (F) are measured in newtons (N) and distances (d) are measured in metres (m)

Each force has a moment, so you will have to calculate moments for EVERY force if more than one force exists, which is usually the case! The size of a moment depends on two things – the size of the force applied and the distance the force acts from the pivot.

This means we first need to scan and locate each force and then worry about the size of the force and the **perpendicular** distance to the pivot in order to calculate the moments. It is very important to remember that the distance from the pivot is measured at a right angle, or perpendicular to the line of action of the force. Let's see how this works. Consider the following diagram. Remember that for each force, we must use the distances based on where the pivot point is.



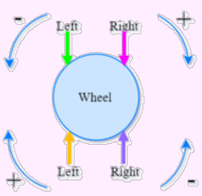
Moment for pink force: $8(3) = 24$

Moment for red force: $4(6) = 24$

Moment for orange force: $4(12) = 48$

We are not yet done though. Each moment has a sign - some moments have a positive sign and some have a negative sign. How do we know which are plus and which are minus? Let's think of a wheel and then ask ourselves what direction certain forces would turn the wheel? Whatever happens to the wheel will happen to the rod/beam! Like with SUVAT we need a positive sense. We take clockwise as our positive sense, so any moment going clockwise will be positive and any moment going anticlockwise will be negative.

If we push where the following forces are below, the wheel will move as follows:



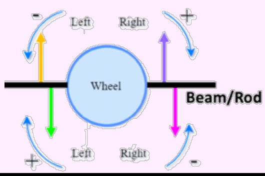
Let's related this to a beam/rod:

If the force is acting downwards on a beam/rod and is on the left side of the pivot ⟹ moves wheel anticlockwise ⟹ −

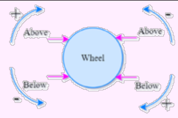
If the force is acting downwards on a beam/rod and is on the right side of the pivot ⟹ moves wheel clockwise ⟹ +

If the force is acting upwards on a beam/rod and is on the left side of the pivot ⟹ moves wheel clockwise ⟹ +

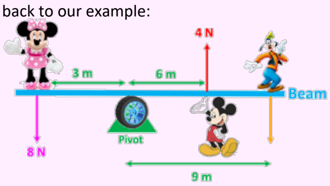
If the force is acting upwards on a beam/rod and is on the right side of the pivot ⟹moves wheel anticlockwise ⟹ −



The signs also work in a similar way if we were to have horizontal forces i.e. forces above and below the pivot. We only have horizontal forces in our example though, so we don't have to worry about this.



So, going back to our example:



The pink force acting downwards on the beam and to the left of the pivot so moment $=-24$
The red force is acting upwards on the beam and to the right of hence so moment $=-24$
The orange force is acting upwards on the beam and to the right of hence so moment $=48$

The principle of moments states that for an object to be balanced, the total clockwise moments must equal the total anticlockwise moments. This is the equivalent of saying the sum of all moments is zero. So, we can sum these forces together (with their relevant signs) and set equal to zero since the rod is in equilibrium. In your course there is always equilibrium, hence this is always the case. Now we have an equation where we can solve for the unknown. Let's check this our example.

Way 1: The sum of all moments is zero

$$+24-24+48=0$$

Way 2: The total clockwise moments must equal the total anticlockwise moments

$$48=24+24$$

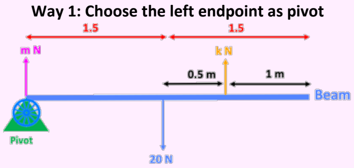
Notice how both agree with the principle of moments since the rod was balanced.

You might be wondering. Can we take any point on the beam as a pivot? Yes, we can use any point on the rod/beam as a pivot point and get the same answer. Also questions often don't give us all the forces. Let's look at an example with an unknown force that needs to be found.

Consider the following uniform beam of length 3 m and weight 20 N with 2 unknown forces $m\ N$ and $k\ N$. The beam is in equilibrium. Find the value of k and the value of m.

This question will be solved with 3 different pivot points chosen in order to illustrate the fact that is doesn't matter which point is chosen to take moments about. We know the weight acts in the middle since the beam is uniform.

Way 1: Choose the left endpoint as pivot

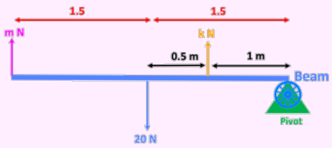


Moment for pink force: $m(0)=0$
Moment for blue force: $20(1.5)=30$
Moment for orange force: $-k(2)=-2k$

The sum of all moments is zero:
$$30-2k=0$$
$$k=15$$

We need to also resolve vertically to find m
$$\uparrow: m-20+k=0$$
$$m+k=20$$
$$m+15=20$$
$$m=5$$

Way 2: Choose the right endpoint as pivot



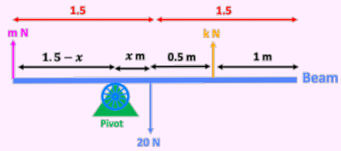
Moment for pink force: $m(3)=3m$
Moment for blue force: $-20(1.5)=-30$
Moment for orange force: $k(1)=k$

The sum of all moments is zero:
$$3m-30+k=0$$
$$k+3m=30$$

We need to also resolve vertically as too many unknowns
$$\uparrow: m-20+k=0$$
$$m+k=20$$

Solve $k+3m=30$ and $m+k=20$ simultaneously
$$m=5, k=15$$

Way 3: Choose a pivot elsewhere



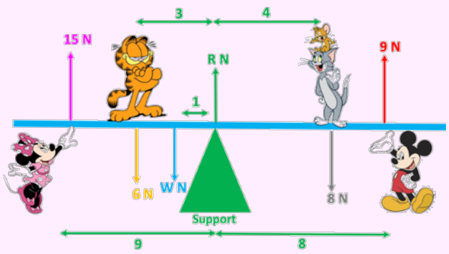
Moment for pink force: $m(1.5-x)=1.5m-mx$
Moment for blue force: $20(x)=20x$
Moment for orange force: $-k(x+0.5)=-kx-0.5k$

The sum of all moments is zero:
$$1.5m-mx+20x-kx-0.5k=0$$
$$1.5m-(m+k)x+20x-0.5k=0 \quad (1)$$

We need to also resolve vertically as too many unknowns
$$\uparrow: m-20+k=0$$
$$k=20-m \quad (2)$$
Sub (2) into (1): $1.5m-20x+20x-0.5(20-m)=0$
$$2m-10=0$$
$$m=5$$
$$k=20-5=15$$

Notice how way 1 was the quickest. This is because choosing the pivot to be where one of the unknown forces was got rid of the unknown m immediately when taking moments since the distance from the pivot to the force was zero i.e. when we took moments at the left pivot, the force m, disappeared because the distance to the pivot was 0, so force × distance $=m\times 0=0$.

Let's look at another example with more forces and a support. Here the triangle represents a support, not a pivot. If there is support, then there will be a reaction force existing since the support is in contact with the beam/road.



Moment for pink force: $15(9)=+135$
Moment for orange force: $6(3)=-18$
Moment for blue force coming from the weight of the beam: $W(1)=-W$
Moment for green force: $R(0)=0$ (we can ignore the force at the pivot since moment=0)
Moment for grey force: $8(4)=+32$
Moment for red force: $9(8)=-72$

Let's use the fact that the sum of the moments is zero:
$$135-18-W+32-72=0$$
$$W=77\text{ N}$$

We can also resolve as normal to get R:
$$\uparrow: 15-6-W+R-8+9=0$$
$$15-6-77+R-8+9=0$$
$$R=6\ N$$

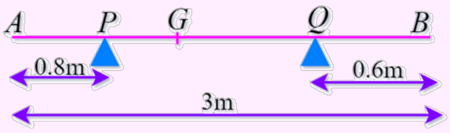
In harder questions more forces might be unknown (such as the rod might have an extra support and hence another reaction force).

Be aware that

- There is never a reaction force to be drawn for a person/object standing on the rod! We only have a reaction force for the supports which are touching the beam.
- The weight force W for the rod/beam only acts in the center of the rod is uniform. If it doesn't act in the centre, then we don't know the distance to each endpoint.

Let's look at a final example:

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where AP = 0.8 m and QB = 0.6 m, as shown in the diagram above. The centre of mass of the rod is at G. Gravity is taken to be 9.8 ms^{-2}.

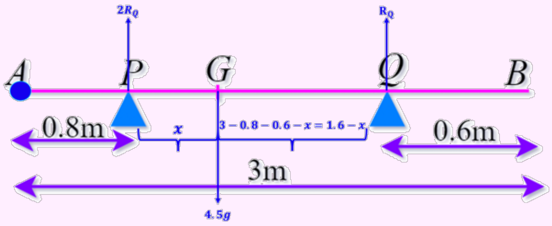


Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

i. the magnitude of the reaction of the support at Q on the rod
ii. the distance AG

i.
Resolve vertically (here is only one unknown reaction force here as a relationship is known between both reaction forces, so we can find by resolving vertically).

$$① \uparrow: 2R_Q - 4.5g + R_Q = 0$$
$$R_Q = 14.7 \text{ N}$$



ii.
we take moments about A since we have already found R_Q and are therefore not worrying about eliminating R_Q (if we were trying to eliminate R_Q we would take moments at the supports P or Q). There are too many distances unknown, so we call any distance x and have the other distance unknown also in terms of x. We call PG x and can then find GQ in terms of x since we know the whole length AB.

$$② M(A) ↻ : -2(14.7)(0.8) + 4.5g(0.8 + x) - 14.7(3 - 0.6) = 0$$
$$x = 0.5333$$
$$\text{AG} = 0.8 + 0.5333 = 1.33$$

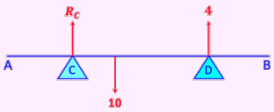
Note: Could have also taken moments about B or P or Q.

For example: $M(B)$ ↻ $: 2(14.7)(3 - 0.8) - 4.5g(1.6 - x + 0.6) + 14.7(0.6) = 0 \Rightarrow x = 0.533$

Step by Step Method With Commonly Asked Questions Answered

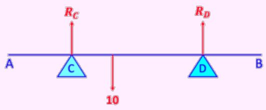
Student often get most confused with which points to use a pivots i.e. which points to take moments about and also what to do if too many distances are unknown or too many forces are unknown. The following will address all of this.

Step 1: Before you even worry about taking moments, resolve vertically as normal (following the template f=ma) to try and find any unknowns if possible. **Students often think that they have too many unknowns because they have forgotten that we can form 2 other equations (1 and 2) above as well as taking moments.**



Here we have vertical forces so we can apply 2) mentioned in the section above.

$$\uparrow: R_C + 4 - 10 = 0$$
$$R_c = 6$$



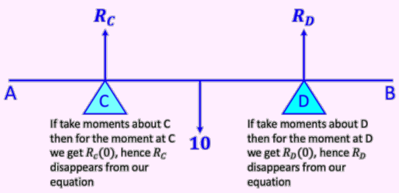
Here we have vertical forces so we can apply 2) mentioned in the section above.

$$\uparrow: R_C + R_D - 10 = 0$$

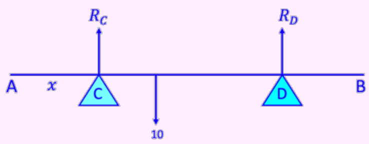
In this case it doesn't help as we still have too many unknowns

Step 2: Now we take moments. We must first **choose a point to take moments about**. A lot of students struggle to know which point to choose to take moments about. The good news is that there is often more than one possible place to choose to be able to answer the question, so it is not always a case of a right or wrong choice. **You can take moments about anywhere, but usually we choose the supports or endpoints**. There are three questions you can ask yourself to help you decide which point would be best, such as:

- Question 1: whether you need to eliminate a certain unknown force? Your main hint to know what point to take moments about is to think about which unknowns you want to get rid of. For example, we decide whether we want to get rid of R_c or R_D in the diagram below. If want to get rid of R_D we take moments about D and if want to get rid of R_C then take moments about C. This is because when you take moments about a point, there is no moment there since the distance to that point is zero, so the force will not feature in your equation.



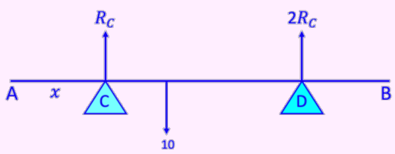
- Question 2: Which point will give the least unknowns and ease of solving for what we want? There is usually no right or wrong point to choose, just usually one of the points makes the algebra easier and takes away the need for simultaneous equations. For example,



  - If we take moments about support C we will have R_D and x unknown
  - If we take moments about support D we will have R_c and x unknown
  - If we take moments about endpoints A or B we will have x unknown and R_c unknown and R_D unknown which is not a good idea.

  Obviously, we have too many unknowns here, so in a question you would be given one of the 3 unknowns or a relationship between the reaction forces. Let's say we were given the value of x. It would be easier to take moments about the pivots rather than the endpoints since taking moments about the pivots would eliminate one of the reaction forces and give us R_c or R_D straight away. We would take moments about pivot C if we wanted R_D and take moments about pivot D if we wanted R_C

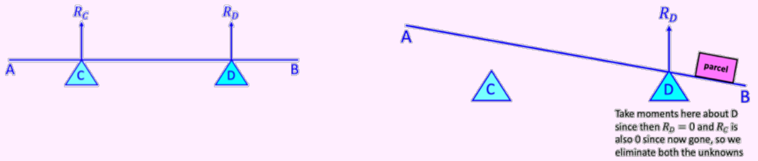
  Let's consider another scenario whereby we're told the magnitude of the reaction of the support at D on the rod is twice the magnitude of the reaction of the support at C on the rod and asked to find the distance x which is AC.



  - To find R_C : Here we know how one reaction force relates to the other and can find R_C straight away just by resolving vertically (there is no need to take moments).
  - To find x : We must take moments and it doesn't matter whether we use the pivots or endpoints since if we moments about endpoints we will have x unknown an R_c unknown and we take moments about either of pivots we will STILL have x unknown and R_c unknown.

  Make sure you have understood the following by this point:
  You calculate the moments about any point whatsoever, and the answers should be consistent. However, as happens pretty often in physics, some choices of framing make the problem easier. Since moment = force x distance, we can take moments about the point where a force is applied so that the distance (and therefore the moment) from that force is 0 in order to simplifying the problem. We can take moments from an endpoint and that will usually either simplify the calculation of distances, or make it easier to calculate the direction of the moments; since forces to the left of the pivot pointing down act opposite forces to the right of the pivot pointing down, it can get confusing to track which moments are clockwise and which are anticlockwise when forces are applied in both directions on both sides of the pivot. By using an endpoint, this problem is partially sidestepped, and their direction is enough to resolve which turning direction they contribute toward.

- Question 3: Does the question tell us that there is a point of tipping? Let's say the beam is at the point of tipping about the point (D) due to a certain load being put on (for instance a parcel). This means that the reaction force at the other point (C) is zero since the beam will not be in contact with C now (right diagram below). Therefore, it would be smart not to use C as a pivot. Instead use point D as the pivot since the reaction force at C is already zero, so we have a chance to eliminate R_D by taking moments about D. R_C and R_D both being gone obviously gives less unknowns which we might want (unless of course you don't want to eliminate them).

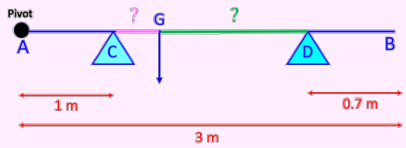


Take moments about D since this will make eliminate R_D and R_C is already 0 so we eliminate both the unknowns at once

Step 3: Scan through and **locate each force** (<u>only consider **forces perpendicular i.e. the vertical forces**</u>)
(Note you will have to deal with diagonal forces i.e. the forces will no longer be only vertical or horizontal. We either break the diagonal force down into horizontal and vertical components first (best method) OR keep it diagonal and use SOHCAHTOA to work out the perpendicular distance from that diagonal force). See cheat sheet Mechanics Inclined Plane.

Step 4: For each of these forces located, we **calculate the moment**. The formula for the moment is the force times the perpendicular distance to point chosen i.e. $M = fd$.
So, we just multiply each of the forces with the **perpendicular** distance to that force from the point chosen.

Note: If struggling with unknown distances to your point chosen as the pivot, don't be afraid to call certain distances x



It appears that don't know the distance CG or GD.

If we choose pivot as A: For the force G we need the distance AG and for the force R_D we need the distance AD. We don't have these. Don't be afraid to call CG x and then we can Write GD in terms of x. $3 - 1 - 0.7 - x = 1.3 - x$

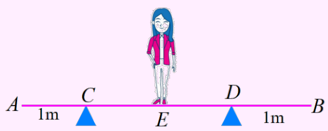
Step 5: Add all the answers for every moment together and solve equal to zero (since in equilibrium) in order to find any unknowns.
Note: an alternative method would be to set the anticlockwise moments equal to the clockwise moments.

Harder Types Of Questions To Be Aware Of

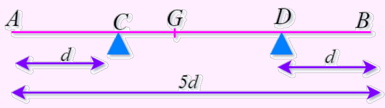
- **Least possible value** of the **coefficient of friction** is basically saying solve F≤ μR
- **Point of tipping** at a point then reaction force at the other point is zero. Take moments about the point where tipping since reaction force at other point is already zero, so have a chance to make reaction at tipping point also 0 by taking moments about it. Reaction force being zero at 2 places obviously give less unknowns which you want!
- **Greatest/Least possible** ⇒ about to tilt about closest pivot point so Reaction force for other pivot point =0

Examples To Try Of Each Type

<u>Example 1:</u>
A uniform beam AB, of mass 40 kg and length 5 m, rests horizontally on supports at C and D, where AC=DB=1m. When a woman of mass 80 kg stands on the beam at E the magnitude of the reaction at D is twice the magnitude of the reaction at C. By modelling the beam as a rod and the woman as a particle, find the distance AE (ans=3.25 m)



<u>Example 2:</u>
A non-uniform rod AB, of mass m and length 5d, rests horizontally in equilibrium on two supports at C and D, where AC = DB = d, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}$m is placed on the rod at B and the rod is on the **point of tipping** about D.

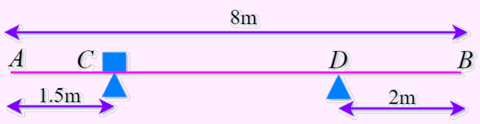


i. Show that GD=$\frac{5}{2}$d

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

ii. Find the magnitude of the normal reaction between the support at D and the rod (ans=$\frac{17}{12}$mg)

A plank AB of mass 20 kg and length 8 m is resting in a horizontal position on two supports at C and D, where AC=1.5mandDB=2m. A package of mass 8 kg is placed on the plank at C, as shown in Figure 2. The plank remains horizontal and in equilibrium. The plank is modelled as a uniform rod and the package is modelled as a particle.



i. Find the magnitude of the normal reaction between the plank and the support at C (ans=166)
ii. Find the magnitude of the normal reaction between the plank and the support at between the plank and the support at D (ans=109)

The package is now moved along the plank to the point E. When the package is at E, the magnitude of the normal reaction between the plank and the support at C is R newtons and the magnitude of the normal reaction between the plank and the support at D is 2R newtons.

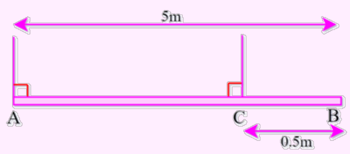
iii. Find the distance AE (ans=5.75m)
iv. State how you have used the fact that the package is modelled as a particle (ans=the weight of the package acts at C or D)

<u>Example 3:</u>
A beam AB has length 5 m and mass 25 kg. The beam is suspended in equilibrium in a horizontal position by two vertical ropes. One rope is attached to the beam at A and the other rope is attached to the point C on the beam where CB = 0.5 m, as shown in Figure 3. A particle P of mass 60 kg is attached to the beam at B and the beam remains in equilibrium in a horizontal position. The beam is modelled as a uniform rod and the ropes are modelled as light strings. Find

i. the tension in the rope attached to the beam at A (ans=43.6N)
ii. the tension in the rope attached to the beam at C. (ans=789.4N)

Particle P is removed and replaced by a particle Q of mass M kg at B. Given that the beam remains in equilibrium in a horizontal position, find

iii. the **greatest possible** value of M (ans=100 kg)
iv. the **greatest possible** tension in the rope attached to the beam at C (ans=125g or 1230N)



<u>Example 4:</u>
A uniform rod AC, of weight W and length $3l$, rests horizontally on two supports, one at A and one at B, where AB = $2l$. A particle of weight 2W is placed on the rod at a distance x from A. The rod remains horizontal and in equilibrium.

i. Find the greatest possible value of x (ans=$\frac{9l}{4}$)

The magnitude of the reaction of the support at A is R. Due to a weakness in the support at A, the **greatest possible** value of R is 2W

ii. find the **least possible** value of x. (ans=$\frac{l}{4}$)